E-ISSN: 2708-4523
P-ISSN: 2708-4515
AJMC 2024; 5(1): 98-105

www.allcommercejournal.com
Received: 19-11-2023
Accepted: 03-01-2024

Oumaima Ben Rebah
Department of Quantitative Methods, Faculty of Economics and Management, University of Sfax, Sfax, Tunisia

Abdelkarim Elloumi
Department of Quantitative Methods, Faculty of Economics and Management, University of Sfax, Sfax, Tunisia

Racem Mellouli
Department of Quantitative Methods, Faculty of Economics and Management, University of Sfax, Sfax, Tunisia

Corresponding Author:
Oumaima Ben Rebah
Department of Quantitative Methods, Faculty of Economics and Management, University of Sfax, Sfax, Tunisia

Advances in cutting & packing problems: A systematic literature review and future directions

Oumaima Ben Rebah, Abdelkarim Elloumi and Racem Mellouli

Abstract
The cutting and packing problem (C&P) is a class of optimization problems that substantially affect resource efficiency in manufacturing and various industries. This review paper comprehensively surveys the state-of-the-art cutting and packing problems (C&P) and their different types and typologies. We explore these challenges' historical evolution and variants, particularly the two-dimensional cutting stock problems. We offer insights into the critical problem formulations and mathematical approaches shaping the field. Additionally, the paper concludes with a discussion of emerging trends, challenges, and potential avenues for future research in the cutting and packing optimization problem.

Keywords: Cutting, packing, two-dimensional, cutting stock, optimization, heuristic, metaheuristic

Introduction

The cutting and packing problems are combinatorial optimization problems (Where the set of feasible solutions is a discrete combinatorial set) at the intersection of mathematical optimization and industrial efficiency. They belong to the class NP [54] and have shown that they are NP-hard in the strong sense. This complex problem arises in industries such as textile, paper, glass, and marble.

Over the years, the cutting and packing problems have garnered significant attention from researchers and practitioners alike. Mathematical formulations, heuristic approaches, and optimization algorithms have been developed to tackle the intricacies of these problems, offering solutions that balance computational efficiency with real-world applicability.

This paper explores the historical evolution of cutting and packing problems, their various types, and the methodologies employed to address their challenges. By understanding the foundations and advancements in this field, we aim to provide valuable insights into the past, present, and future of cutting stock problems. The remainder of the paper is organized as follows: In Section 2, the definition, the relationship, the differences, and the classification of these problems are described. Section 3 gives a detailed review of related works from the literature. A plethora of optimization techniques to tackle the cutting and packing problems, from classic algorithms to more recent metaheuristic approaches, are provided in Section 4. Finally, some conclusions are drawn, including a discussion of new trends, problems, and prospective future research directions in the cutting and packing problems.

A general overview of the cutting and packing problems

Definition & Classification

The cutting and packing problems are mathematical optimization problems. They were first studied by [50], who aimed to model them suitably [36]. Carried on with this work to solve some of its variants and generalize it to two-dimensional cutting problems.

It consists in determining how, in a "material" of given dimensions, to cut (Or arrange) a maximum number of elements of smaller dimensions and, again, how to cut (Or arrange) a given number of elements in such a way as to use a minimum quantity of the primary "material."

It can also correspond to placing a set of small objects (Items) in a set of large objects (Bags or bins) in such a way that the total surface used is the smallest possible or that the total profit of the placed objects is within the maximum claimed in the problem formulation. It can be defined by finding the best assortment of small objects (Items) in large objects (Bins) that minimize material waste while satisfying all problem constraints.

To describe the varieties of the cutting and packaging problem and to define the constraints for each type of problem, it is required to go through a classification, which not only standardizes the definitions and notations but also facilitates communication between researchers in this domain. The classification of cutting and packing problems is further specified by several typologies that offer a good tool for organizing and categorizing the existing literature [24].

Introduces the first typology. This typology considers four parameters with different possible cases that identify 96 problems.

- The first parameter is used to identify the dimension of the problem.
- The second parameter informs about the type of assignment, i.e., whether all components should be placed inside the container(s) or only a part should be positioned. In the first case, the problem is a minimization performed on the containers (size or number). In the second case, a maximization is performed on the components to be positioned. Concretely, it will be a question of maximizing the used space or a profit function by selecting a subset of the components.
- The third parameter provides information about the type of container.
- The fourth parameter contains information on the nature of the components, i.e., whether they are identical, slightly different, or strongly different.

Table 1: Classification proposed by Dyckhoff [24]

Dimension of the problem	Assignment types	Nature of the containers	Nature of the components
1: One-dimensional problem; 2: Two-dimensional problem; 3: Three-dimensional problem; N: Multi-dimensional problem with N > 3.	B: All containers and a few components; V: A set of containers and all components.	O: One single container; I: Several identical containers; D: Several different containers.	F: Some components; M: Many components which are different from each other; R: Many components are quite similar to each other; C: Identical components.

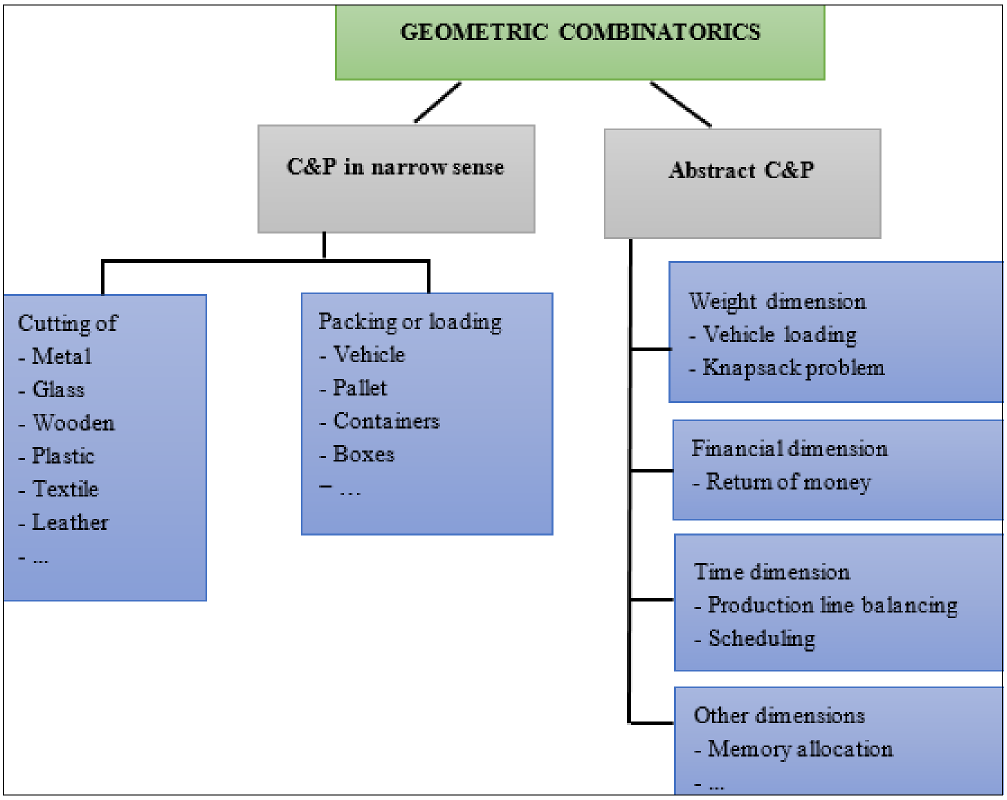

Fig 1: The phenomenology of cutting and packaging problems proposed by Dyckhoff [24].

Then, in 1992 [25] as well [23] provided an overview of cutting and packing problems, and in 1997 [26] published an annotated bibliography of cutting and packing problems.

In addition, other authors [42] and [56] have presented specific typologies for the two-dimension cutting problems.

However, over time and recent developments [65] have been inspired by Dyckoff's [24] typology, a new classification adapted to the evolution of the cutting and packing problems.

From this improved classification, the main types of problems (Cutting and packing) have been defined, which are developed by the combination of the following characteristics:

- The objective of the problem, i.e., a maximization of the components or minimization to be performed on the container.
- The second and third characteristics concern the container's geometry and the components.

From these three characteristics, various types of problems are established. Each of these problems can be refined based on the specificities of each problem.

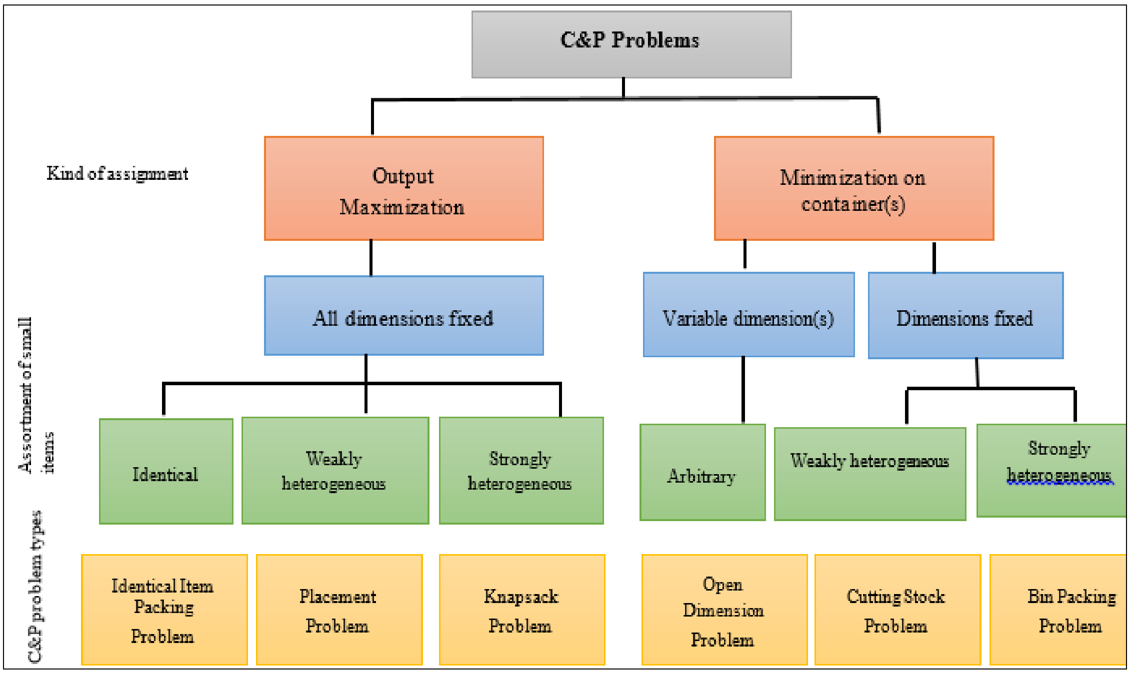

Fig 2: Classification according to Wäscher *et al.*, [56].

Relation between Cutting and Packing problems

According to [24], the relationship between cutting and packing problems can be explained as follows:

The cutting problems can be considered as placing elements in large objects (plates or strips). Similarly, packing problems can be considered as cutting problems. This is where the compound term "Cutting and Packing" came from.

Further, [29] explained that this strong relationship between the C & P problems results from the duality of a material body and its space. Indeed, packing a list of boxes into a container is the same as cutting the container's space, producing pieces where it is possible to place the boxes. Conversely, cutting a sheet of material to obtain smaller items is the same as packing the items into the sheet.

The most known variants of these problems are mentioned below

The Knapsack problem: This problem models a situation analogous to filling a Knapsack, which cannot support more than a certain weight, with all or part of a given set of objects, each having a weight and a value.

Two types of this problem can be distinguished

- The unidimensional-Knapsack where the problem consists of selecting a set of components, each having a weight and a profit, to maximize the total profit while respecting a maximum weight constraint. Each component is associated with a binary variable that models the selection or not of the component in the knapsack.
- The multi-dimensional knapsack problem can be divided into 2D and 3D. In 2D, the components and the container are rectangles aligned on the axis system. In 3D, the components are rectangular parallelepipeds.

The bin packing problem: The bin packing problem generally consists of finding the most economical arrangement of a (Weakly heterogeneous) set of small objects (Items) in a given set of limited and voluminous objects (Bins).

Thus, bin-packing problems consist of placing items in one or more bins characterized by shape. The variants are distinguished according to the size, the a priori knowledge of the items, the shape of the items and the bins (square, rectangular, circular), and the possibility to modify the orientation of the items.

The cutting stock problem: This problem has been widely studied in the literature with numerous practical cases, such as in the paper, furniture, and glass industries. The domain of application is different, but the question of determining how to cut a set of small objects (items) according to the required backlog from large objects (bins) available in stock, which can be homogeneous or heterogeneous, these problems include two sub-problems:

- Determine the objects from stock to be used to satisfy the order.
- Find the best pattern that minimizes the trim loss.

The strip packing: This problem is a 2-dimensional geometric minimization problem that consists of finding a feasible configuration composed of all the elements that minimize the area occupied in the strip. In this problem, we can have regular or irregular shapes of the elements. Given a set of rectangles aligned on the axis and a strip of finite width and infinite height, determine a non-overlapping packing of the rectangles in the strip by minimizing its height.

The difference in complexity between C&P problems is due to their constraints, which depend on the objects' characteristics to be packed or cut and the cutting tools. These constraints are described as follows:

Geometric constraint: The material is one of the essential elements of the cutting problem, and they differ by their Shapes, Homogeneity, and Dissymmetry.

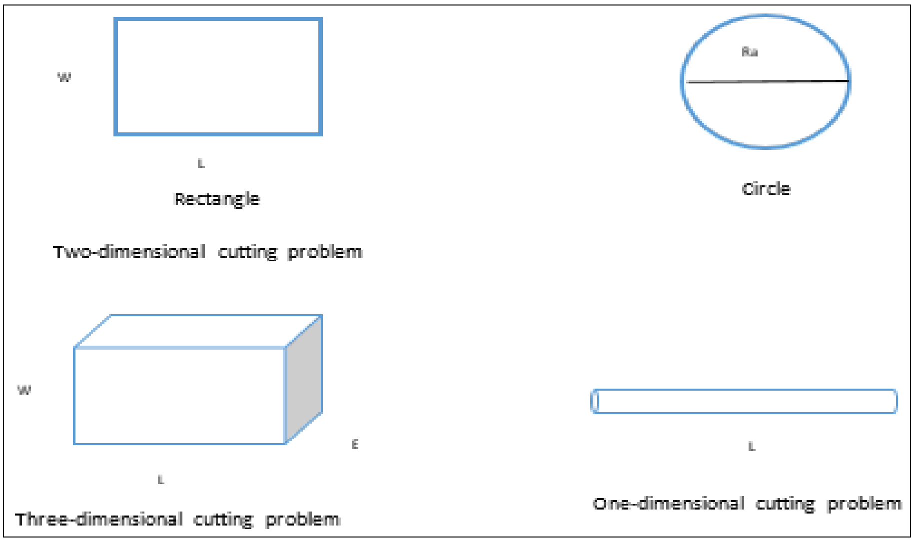

Fig 3: The various types of materials.

Rotation constraint: An oriented cut is considered if the material to be cut imposes an arrangement of the pieces in a specific orientation (The rotation is prohibited). In the other case of the non-oriented cut, the orientations are allowed, and the parts can be rotated by 90 degrees.

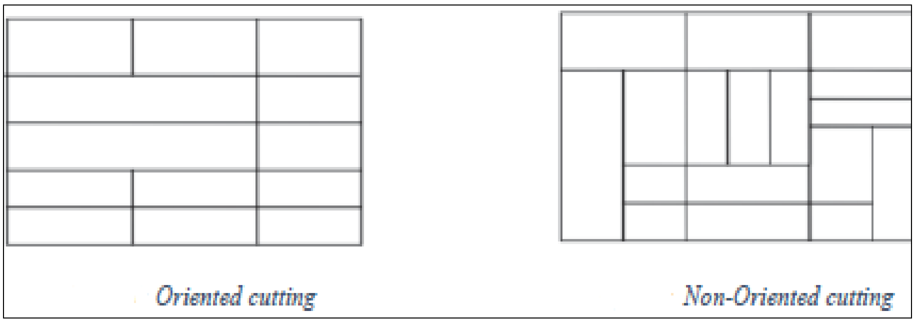

Fig 4: Oriented and Non-Oriented cutting.

Cutting tools constraint

- **Guillotine cutting:** If we assume that the material is a rectangular plate, the cutting is done by dissecting from one side to its affixed parallel to the other two.
- **Non-guillotine cutting:** Generally, this cut generates a better solution than guillotine-type cuts. This cut consists of using the same process as in the guillotine cut, and it can be carried out while marking stops to reach the opposite side of the (sub) rectangle to be cut.
- **Non-orthogonal cutting:** This cut does not consider the parts' orientation. In this case, the parts can be rotated and translated (they are rotated, so they are not fixed).

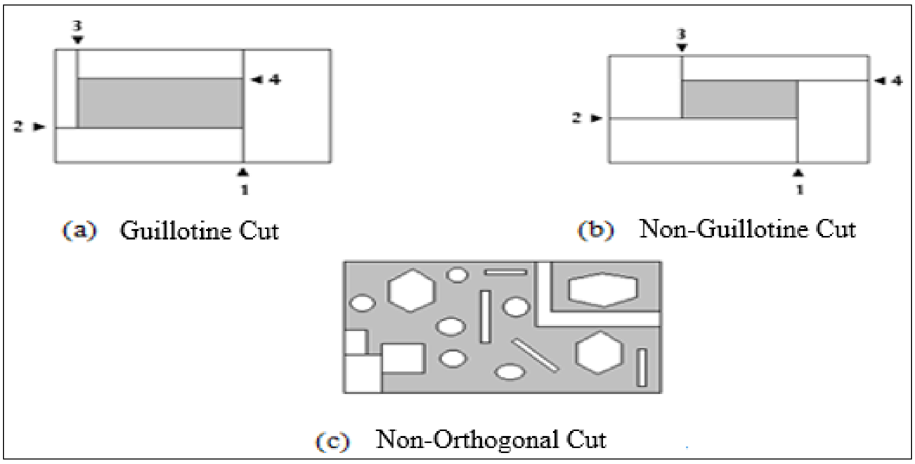

Fig 5: The different types of cut.

Related works

The cutting problem has been studied extensively for over 50 years. According to history, [50] was the first to model the cutting problem [35]. Did some of the first works on one- and two-dimensional inventory-cutting problems. They showed that they could be formulated as a linear programming problem, and the one-dimensional case can be solved using a knapsack function at each pivot. The same authors [36-37] generalized their model to multidimensional-cutting problems relying on other exact solution techniques. Later, [18] used the cutting problem to study multiprogramming systems [33], used it to investigate multiprocessor systems.

The search for a cutting problem solution differs according to the guillotine, non-guillotine, or non-orthogonal tools. The guillotine cut (edge-to-edge cut) is a straight bisecting line going from one edge of an existing rectangle to the opposite edge. Note that non-guillotine cuts (with possible stop markings without reaching the opposite sides) generate better solutions than guillotine ones. The non-orthogonal cuts do not consider the orientation of the parts (possible rotation of the cutting pattern).

Many authors have been interested in the guillotine context, for example [35-36-37, 38-39-61, 41, 1-2, 15, 64, 7, 62, 30, 63, 13, 31, 66] Etc.

Others have studied the problem using a non-guillotine tool, for example, [8, 21] Etc.

Concerning the two-dimensional stock cutting problems, and since [37, 41] was the first to contribute with a more efficient computational procedure using a recursive algorithm where cuts are performed with a guillotine tool. Soon after [17], used an enumerative tree search algorithm with each node representing a cut to solve the problem while maintaining the guillotine restriction [43]. Improved the algorithm of [63] by using the properties of a dynamic programming solution.

Without the guillotine constraint, dynamic programming approaches were proposed by [44-46-47], column generation by [59, 60], and integer linear programming formulations by [55, 11] developed a branch and cut and price algorithm.

Concerning approximate methods [34], proposed a bottom-left heuristic based on the first-fit strategy [12]. Proposed a generalization of the first-fit algorithm that they call Finite First Fit (FFF) [56]. Suggested the Alternate Directions heuristic (AD) based on the principle of the Floor-Ceiling (FC) method. When a level becomes full, the latter allows turning it over, exchanging the top and bottom, the left and right, and then arranging objects on the left again [46]. Solved the two-dimensional cutting problem using a hybrid approach that combines two heuristics, a depth-first search on a search tree using a hill-climbing technique and a dynamic programming procedure based on a solution of a series of one-dimensional bag-to-bag problems [47]. Combined a gluttonous algorithm and a hill-climbing strategy [3]. Proposed an approach that combines the Path Relinking method, strip construction, and a GRASP strategy [28]. Developed an approach compounding a heuristic metric and the maximal items area concept. In addition, varieties of metaheuristics have been adopted, for example, simulated annealing by [52], genetic algorithm [9], and Tabu search [3].

Resolution Methods

Most C&P problems require heuristic or metaheuristic approaches to be solved because of the complexity of these problems, which belong to the NP-hard class. However, this does not prevent the possibility of solving them by exact methods when the considered problem includes a small instance.

The first integer linear programming formulation of the one-dimensional cutting problem [35] and the 2-D cutting problem with constraint levels [36] was given by Gilmore and Gomory-otherwise, [55]. The main feature of this model is the explicit verification of the guillotine constraint by distinguishing between parts that initialize a strip (The first part of a strip) and parts that are cut from a strip. Indeed, many algorithms for solving linear programs in cases where the variables used are real. We cite the simplex method as an example.

The Branch and Bound (B&B) algorithm [22] is used to solve combinatorial optimization problems with a large number of possible solutions and, in particular, to solve integer linear programming problems where the notion of a feasible solution (satisfying the constraints) can be defined. It is based on a tree approach of searching for an optimal solution by separations and evaluations, where a tree of states, with nodes and leaves, represents the solution. Among the authors who use this method [7, 32, 10, 51].

This method is based on three main axes

- **Evaluation:** This reduces the search space by eliminating subsets that do not contain the optimal solution.
- **Separation:** This consists of dividing the problem into sub-problems.

The path strategy includes three types of strategies

Width-first: This strategy favors the vertices closest to the root by making fewer separations from the initial problem.

Depth-first: This strategy favors the vertices farthest from the root (Of higher depth) by applying more separations to the initial problem. This path quickly leads to an optimal solution by saving memory.

The best first: This strategy explores sub-problems with the best bound. It also avoids exploring all sub-problems with a wrong evaluation of the optimal value.

Moreover, the dynamic programming approach divides the initial problem into small sub-problems. Evaluating these sub-problems eliminates the less interesting ones from the search space. Therefore, dynamic programming techniques can find a succession of decision coordinates to reach the optimal solution [36] were the first to use dynamic programming in a recursive function for the two-dimensional cutting problem. Later, [6] improved this formulation to solve the unconstrained guillotine-cutting problem. Also, [16, 45] have used this approach to solve the cutting problem.

For heuristics resolution methods, we distinguish between two categories for C&P problems: one-phase methods and two-phase methods.

The one-phase methods are based on the iterative arrangement of objects in the bins.

There are rules to be defined

- The order in which the objects are examined.
- The bin.
- The position in which one seeks to place them first.

In addition, most approaches are layer algorithms. The pieces are positioned from left to right to form the layers. The first layer corresponds to the bottom of the bin (Strip), and the layers are created successively by a horizontal line that coincides with the top of the highest piece placed on the plate below [19] suggested a bottom-left (BL) heuristic based on the first-fit strategy for the inventory reduction problem. Initially, only one bin is considered when there is no more space to store the current object in the first bin. A second bin is opened but without closing the first one. In an intermediate step where there are *k* open bins numbered from 1 to k according to the order of their first use, the current object *i* is stored in the lowest numbered bin that can contain it. Suppose no bin can contain *i,* a new *k + 1* bin is created without closing the others. The order in which the objects are processed is crucial for the quality of the solution.

This algorithm belongs to the family of heuristics that preserve the bottom-left stability condition. A rectangle preserves the bottom-left stability condition only if positioned on the lowest and leftmost empty surface. At each iteration of the algorithm, each bin keeps a list of maximal, in terms of inclusion, empty surfaces. The BL heuristic proceeds by placing the current object in the lowest and leftmost surface in the first bin that can hold it. The process is then repeated as long as objects still need to be placed.

In addition, [12] have also proposed a generalization of the First-Fit algorithm, which they call "Finite First Fit (FFF)."

Moreover, the "Alternate Directions (AD)" heuristic is based on the principle of the Floor-Ceiling (FC) method, which was proposed by [56]. When a level becomes full, this method allows turning it upside down, exchanging the top and the bottom, the left and the right, and then arranging objects on the left again. This heuristic works in two steps:

In the first step, the algorithm initializes with a number of bins equal to a lower bound on the number of bins needed. Then, it starts by placing objects at the bottom of these bins according to the BFD (Best Fit Decreasing) rule, which applies the same strategy as the first-fit rule, keeping the bins open. However, the choice of the bin in which the current object i will be placed depends on the values of the gaps (Free spaces) in the bins. Thus, i will be placed in the bin with the smallest gap that can hold it. While in the second step, the remaining objects are arranged in strips alternately from left to right and from right to left. When an object cannot be placed in any direction in the current bin, it is assigned to the next initialized bin, or a new bin is created.

The "HBP" heuristic was proposed [14]. The idea of this heuristic is to rerun an algorithm several times that processes the objects in an order fixed at the beginning of each rerun. Only one bin is considered at a time. When no more objects can be placed in the current bin, we close it, and it will not be considered again. In this case, a new bin is opened, and the operation is repeated until all the objects are placed.

On the other hand, the "Item Maximal Area (IMA)" heuristic proposed by [28], contrary to classical heuristics, which consist of allocating objects to bins in a predefined order, does not require any pre-ordering of objects. Instead, we look for the best pair (Object, bin) among all feasible pairs at each step of ordering an object. The couple's choice is managed by a specific criterion, which depends on the characteristics of the objects and the considered surfaces.

For the two-phase methods, in the first phase, a solution is sought for the following strip-packing problem: the set of objects is the original set *I*, and the bin width is equal to *W*. Next, a solution is determined for the original two-dimensional stock cutting problem by cutting the resulting strip into bins of height *H*.

The following methods differ in the algorithms used in the two phases. In most cases, the first algorithm arranges the objects by levels. For example, when an object *i* is arranged on the left, we try to arrange the following objects in the horizontal band on the right of *i*. This method allows the management of only bands of width *W* and variable height in the second step, thus treating it as a one-dimensional cutting problem.

Moreover, some algorithms use heuristics for the one-dimensional cutting problem during the first phase. Among these algorithms, we mention the "Hybrid Best Fit method" (HBF) proposed by [12].

Again, some methods are also based on the transformation of the problem into a bag-to-bag problem, such as the "Knapsack packing (KP) method" by [56]. The levels created are filled to the maximum using the one-dimensional knapsack solution algorithm.

Among the used metaheuristics in the literature to solve the problem of (C&P), we distinguish:

The genetic algorithm: Search metaheuristics inspired by Charles Darwin's theory of natural evolution. This algorithm reflects the process of natural selection, where the fittest individuals are selected for reproduction to produce the next generation's offspring.

Among the authors who used this algorithm are [49, 4, 57].

GRASP: The gluttonous random adaptive search procedure was proposed by [40] and generally involved iterations consisting of successive constructions of a gluttonous random solution and subsequent iterative improvements to it through local search. The greedy randomized solutions are generated by adding elements to the problem's solution set from a list of elements ranked by a greedy function according to the solution quality they will achieve. Many authors have used this approach to solve C&P problems such as [20, 58].

Simulated annealing: Is a metaheuristic to approximate global optimization in a large search space for an optimization problem. It is often used when the search space is discrete. This metaheuristic does not search for the best solution in the neighborhood of the current solution. Instead, one draws at random a solution from the neighborhood. If the solution is better, it is always accepted as a new current solution, but if the solution is worse than the present current solution is accepted with a certain probability. Among the authors who used this algorithm to solve the C&P problems, we mention [53, 48].

Particle swarm algorithm: Is a computational method that optimizes a problem by iteratively trying to improve a candidate solution about a given quality measure. It solves a problem by having a population of candidate solutions, here dubbed particles, and moving them around in the search space according to a simple mathematical formula over the particle's position and velocity. Each particle's movement is influenced by its local best-known position. However, it is also guided toward the best-known positions in the search space, updated as other particles find better positions. We can mention some authors who used this approach [27, 5].

Conclusion

In conclusion, this review paper has delved into the multifaceted realm of cutting and packing problems, in particular the two-dimensional cutting stock problem, unravelling the intricate challenges and diverse methodologies employed to address them. By tracing the historical evolution and exploring and analyzing solution approaches.

The cutting and packing problem, with its various incarnations, poses a substantial challenge in operational research and combinatorial optimization. The literature review has underscored the efforts of researchers to devise efficient algorithms while considering the NP-hard nature of these problems and the practical constraints encountered in real-world applications.

Looking to the future, several perspectives and directions emerge for further exploration. It's interesting to integrate advanced technologies, such as artificial intelligence, machine learning, and Industry 4.0 principles, that hold promise for enhancing the efficiency and adaptability of cutting and packing algorithms in dynamic manufacturing environments.

In addition, future research should focus more on incorporating sustainability metrics into cutting and packing optimization, aligning with the growing emphasis on eco-friendly practices and minimizing waste in production processes. Also, tailoring cutting and packing solutions to specific industries or production scenarios can enhance the applicability and effectiveness of optimization algorithms. Furthermore, the collaboration between researchers from operational research, computer science, and manufacturing industries can facilitate the cross-pollination of ideas and lead to innovative, holistic solutions.

References

1. Adamowicz M, Albano A. Nesting two-dimensional shapes in rectangular modules. Computer-Aided Design. 1976;8(1):27-33.
2. Adamowicz M, Albano A. Two-stage solution of the cutting stock problem. In: IFIP Congress. Vol 2. North-Holland; 1971:1086-1091.
3. Alvarez-Valdes R, Martí R, Tamarit J, Parajón A. GRASP and Path Relinking for the Two-Dimensional Two-Stage Cutting-Stock Problem. INFORMS Journal on Computing. 2007;19(2):261-272.
4. Anand K, Babu A. Heuristic and genetic approach for nesting of two-dimensional rectangular shaped parts with common cutting edge concept for laser cutting and profile blanking processes. Computers & Industrial Engineering. 2015;80:111-124.
5. Ayadi O, Barkallah M. An adapted particle swarm optimization approach for a 2D guillotine cutting stock problem. Mechanics & Industry. 2016;17(5):508.
6. Beasley J. Algorithms for unconstrained two-dimensional guillotine cutting. Journal of the Operational Research Society. 1985;36(4):297-306.
7. Beasley J. An algorithm for the two-dimensional assortment problem. European Journal of Operational Research. 1985;19(2):253-261.
8. Beasley J. An Exact Two-Dimensional Non-Guillotine Cutting Tree Search Procedure. Operations Research. 1985;33(1):49-64.
9. Beasley J. A population heuristic for constrained two-dimensional non-guillotine cutting. European Journal of Operational Research. 2004;156(3):601-627.
10. Bekrar A. Résolution du problème de placement en deux dimensions: Heuristiques, bornes inférieures et méthodes exactes [Doctoral dissertation]. Troyes; c2007.
11. Belov G, Scheithauer G. A branch-and-cut-and-price algorithm for one-dimensional stock cutting and two-dimensional two-stage cutting. European Journal of Operational Research. 2006;171(1):85-106.
12. Berkey J, Wang P. Two-Dimensional Finite Bin-Packing Algorithms. Journal of the Operational Research Society. 1987;38(5):423-429.
13. Blazewicz J, Drozdowski M, Soniewicki B, Walkowiak R. Two-Dimensional Cutting Problem. International Inst. Appl. Sys. Analysis; c1991. p. 1-39.
14. Boschetti M, Mingozzi A. The two-dimensional finite bin packing problem. Part II: New lower and upper bounds. Quarterly Journal of the Belgian, French and Italian Operations Research Societies. 2003;1(2):135-147.
15. Cani P. A Note on the Two-Dimensional Rectangular Cutting-Stock Problem. Journal of the Operational Research Society. 1978;29(7):703-706.
16. Christofides N, Hadjiconstantinou E. An exact algorithm for orthogonal 2-D cutting problems using guillotine cuts. European Journal of Operational Research. 1995;83(1):21-38.
17. Christofides N, Whitlock C. An Algorithm for Two-Dimensional Cutting Problems. Operations Research. 1977;25(1):30-44.
18. Codd E. Multiprogram scheduling: parts 1 and 2. introduction and theory. Communications of the ACM. 1960;3(6):347-350.
19. Coffman E, Garey M, Johnson D. Approximation algorithms for bin-packing-an updated survey. In: Algorithm design for computer system design. Springer, Vienna; c1984. p. 49-106.
20. Da Silveira J, Miyazawa F, Xavier E. Heuristics for the strip packing problem with unloading constraints. Computers & Operations Research. 2013;40(4):991-1003.
21. Daniels J, Ghandforoush P. An Improved Algorithm for the Non-Guillotine-Constrained Cutting-Stock Problem. Journal of the Operational Research Society. 1990;41(2):141-149.
22. Douiri M, Elbernoussi S, Lakhbab H. Cours des méthodes de résolution exactes heuristiques et métaheuristiques. Université Mohamed V, Faculté des sciences de Rabat; c2009. p. 5-87.
23. Dowsland K, Dowsland W. Packing problems. European journal of operational research. 1992;56(1):2-14.
24. Dyckhoff H. A typology of cutting and packing problems. European Journal of Operational Research. 1990;44(2):145-159.
25. Dyckhoff H, Finke U. Cutting and packing in production and distribution: A typology and bibliography. Springer Science & Business Media; c1992.
26. Dyckhoff H, Scheithauer G, Terno J. Cutting and packing. Annotated bibliographies in combinatorial optimization; c1997. p. 393-412.
27. Eberhart R, Kennedy J. A new optimizer using particle swarm theory. In: MHS'95. Proceedings of the sixth international symposium on micro machine and human science; c1995. p. 39-43.
28. El Hayek J, Moukrim A, Negre S. New resolution algorithm and pretreatments for the two-dimensional

bin-packing problem. Computers & Operations Research. 2008;35(10):3184-3201.
29. Faina L. A survey on the cutting and packing problems. Bollettino dell'Unione Matematica Italiana. 2020;13(4):567-572.
30. Farley A. Selection of stock plate characteristics and cutting style for two-dimensional cutting stock situations. European Journal of Operational Research. 1990;44(2):239-246.
31. Fayard D, Zissimopoulos V. An approximation algorithm for solving unconstrained two-dimensional knapsack problems. European Journal of Operational Research. 1995;84(3):618-632.
32. Fekete S, Schepers J, Van Der Veen J. An exact algorithm for higher-dimensional orthogonal packing. Operations Research. 2007;55(3):569-587.
33. Garey M, Graham R. Bounds for Multiprocessor Scheduling with Resource Constraints. SIAM Journal on Computing. 1975;4(2):187-200.
34. Garey M, Johnson D, Coffman E. Approximation Algorithms for Bin-Packing - An Updated Survey. In: Algorithm design for computer system design. Vienna: Springer; c1984. p. 49-106.
35. Gilmore G, Gomory R. A linear programming approach to the cutting-stock problem. Oper. Res. 1961;9(6):849-859.
36. Gilmore P, Gomory R. Multistage cutting stock problems of two and more dimensions. Operations research. 1965;13(1):94-120.
37. Gilmore P, Gomory R. The theory and computation of knapsack functions. Operations Research. 1966;14(6):1045-1074.
38. Haessler R. Controlling Cutting Pattern Changes in One-Dimensional Trim Problems. Operations Research. 1975;23(3):483-493.
39. Haessler R, Sweeney P. Cutting stock problems and solution procedures. European Journal of Operational Research. 1991;54(2):141-150.
40. Hart J, Shogan A. Semi-greedy heuristics: An empirical study. Operations Research Letters. 1987;6(3):107-114.
41. Herz J. Recursive Computational Procedure for Two-dimensional Stock Cutting. IBM Journal of Research and Development. 1972;16(5):462-469.
42. Hifi M. Rapport d'habilitation à diriger les recherches; c1998.
43. Hifi M. An improvement of viswanathan and bagchi's exact algorithm for constrained two-dimensional cutting stock. Computers & Operations Research. 1997;24(8):727-736.
44. Hifi M, M'Hallah R. An Exact Algorithm for Constrained Two-Dimensional Two-Staged Cutting Problems. Operations Research. 2005;53(1):140-150.
45. Hifi M, Ouafi R. Best-first search and dynamic programming methods for cutting problems: The cases of one or more stock plates. Computers & industrial engineering. 1997;32(1):187-205.
46. Hifi M, Roucairol C. Approximate and exact algorithms for constrained (Un) weighted two-dimensional two-staged cutting stock problems. Journal of combinatorial optimization. 2001;5(4):465-494.
47. Hifi M, M'Hallah R, Saadi T. Strip generation algorithms for constrained two-dimensional two-staged cutting problems. European Journal of Operational Research. 2006;172(2):515-527.
48. Hong S, Zhang D, Lau H, Zeng X, Si Y. A hybrid heuristic algorithm for the 2D variable-sized bin packing problem. European Journal of Operational Research. 2014;238(1):95-103.
49. Hopper E, Turton B. An empirical investigation of meta-heuristic and heuristic algorithms for a 2D packing problem. European Journal of Operational Research. 2001;128(1):34-57.
50. Kantorovich L. Mathematical methods of organizing and planning production. Management Science. 1960;6(4):366-422.
51. Kenmochi M, Imamichi T, Nonobe K, Yagiura M, Nagamochi H. Exact algorithms for the two-dimensional strip packing problem with and without rotations. Eur J Oper Res. 2009;198(1):73-83.
52. Lai K, Chan J. Developing a simulated annealing algorithm for the cutting stock problem. Comput Ind Eng. 1997;32(1):115-127.
53. Leung S, Zhang D, Sim K. A two-stage intelligent search algorithm for the two-dimensional strip packing problem. Eur J Oper Res. 2011;215(1):57-69.
54. Lewis H, Garey M, Johnson D. Computers and intractability. A guide to the theory of NP-completeness. WH Freeman and Company, San Francisco. 1979;338:498-500.
55. Lodi A, Monaci M. Integer linear programming models for 2-staged two-dimensional knapsack problems. Math Program. 2003;94(2):257-278.
56. Lodi A, Martello S, Vigo D. Heuristic and metaheuristic approaches for a class of two-dimensional bin packing problems. INFORMS J Comput. 1999;11(4):345-357.
57. Lu H, Huang Y. An efficient genetic algorithm with a corner space algorithm for a cutting stock problem in the TFT-LCD industry. Eur J Oper Res. 2015;246(1):51-65.
58. MirHassani S, Jalaeian Bashirzadeh A. A GRASP meta-heuristic for two-dimensional irregular cutting stock problem. Int. J Adv Manuf Technol. 2015;81(1):455-464.
59. Morabito R, Garcia V. The cutting stock problem in a hardboard industry: A case study. Comput Oper Res. 1998;25(6):469-485.
60. Song X, Bennell J. Column generation and sequential heuristic procedure for solving an irregular shape cutting stock problem. J Oper Res Soc. 2014;65(7):1037-1052.
61. Sweeney P, Haessler R. One-dimensional cutting stock decisions for rolls with multiple quality grades; c1988, 44(2).
62. Vasko F. A computational improvement to Wang's two-dimensional cutting stock algorithm. Comput Ind Eng. 1989;16(1):109-115.
63. Viswanathan K, Bagchi A. Best-First Search Methods for Constrained Two-Dimensional Cutting Stock Problems. Oper Res. 1993;41(4):768-776.
64. Wang P. Two Algorithms for Constrained Two-Dimensional Cutting Stock Problems. Oper Res. 1983;31(3):573-586.
65. Wäscher G, Haußner H, Schumann H. An improved typology of cutting and packing problems. Eur J Oper Res. 2007;183(3):1109-1130.
66. Zissimopoulos V. Heuristic methods for solving (Un) constrained two-dimensional cutting stock problems. Methods Oper Res. 1985;49:345-357.